Refraction and reflection of a nonrelativistic wave when the interface and the media are moving

A. Ascoli,[1] C. Bernasconi,[2] and G. Cavalleri[3]
[1]*Dipartimento di Scienze Molecolari Agroalimentari, Università di Milano, Via Celoria 2, Milano, Italy*
[2]*Via Castelbarco 22, Milano, Italy*
[3]*Università Cattolica, Via Trieste 17, Brescia, Italy*
(Received 26 March 1996)

Generalized Snell-Descartes expressions are given in vector form for the refraction and reflection of a classical sound wave when the two supporting fluids and their interface are in motion with nonrelativistic speeds. The validity of Fermat's principle is extended to fluids at rest, separated by a moving interface. [S1063-651X(96)14211-2]

PACS number(s): 43.20.+g, 43.28.+h

I. INTRODUCTION

The problem in the title could have been solved in a general manner already in the past century. However, we have not found its general solution in recent specialized reviews and treatises [1]. Indeed, the problem has interested several workers in the past, but none of them obtained real general results. We survey their works in Sec. II.

Actually we need the solution of this problem for a nonrelativistic model of elementary particles and also for the classical interpretation of the Arago and the Michelson-Morley experiment. We therefore solve here the most general case when the surface separating two different fluids has a velocity $\mathbf{V}$, the fluid in one half space has a nonrelativistic velocity $\mathbf{u}_1$, and the fluid in the other half space has a nonrelativistic velocity $\mathbf{u}_2$. Refraction is what we need for our future research. The reflection is also given for generality.

At first sight, it could appear strange to have three independent velocities, namely $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{V}$, since $\mathbf{u}_2-\mathbf{u}_1$ must be tangential to the bimaterial interface if the latter remains intact, i.e., if it acts as an impenetrable barrier without sources, wells, and pores. As said, the general case of three independent velocities arose during the elaboration of a new, nonrelativistic model of elementary particles conceived as local concentrations of the ether. Such ether-concentration particles are therefore thought of as extended (not pointlike). Within one of these extended particles, let us consider an ether isodensity surface σ (hereafter ''the interface''). If the particle is in a steady-state motion with velocity $\mathbf{V}_p$, and if $\hat{\mathbf{n}}$ is the unit vector perpendicular to σ in a point P of it, it is $\mathbf{V}=\mathbf{V}_p\cdot\hat{\mathbf{n}}\hat{\mathbf{n}}$. The ether flows inside the particle through its forward front and escapes from it through its rear part. The two neighborhoods of P on either side of σ have different velocities $\mathbf{u}_1$ and $\mathbf{u}_2$ with $\mathbf{u}_2-\mathbf{u}_1$ being, in general, not tangential to the interface.

The most similar macroscopic example is that of a cylinder in which a porous piston moves with $\mathbf{V}$ and keeps two different pressures in the two parts containing the same gas. The densities, hence the speeds of sound, of the two parts are different and the diffusion of the gas contained in the part at higher pressure into the other part at a lower pressure implies $\mathbf{u}_2-\mathbf{u}_1$ perpendicular to the piston surface (acting as an interface).

The familiar case of $\mathbf{u}_2-\mathbf{u}_1$ tangential to the interface is a particular case of the general one treated in this paper. Not even this less general case has been found by us in the literature. If other solutions have escaped our bibliographical research (Sec. II), ours is still worth publishing since no trace of the others appears in recent literature.

II. LITERATURE SURVEY

Only the very particular case of the refraction of sound through a fixed interface separating the same fluid in two parts, the fluid in one part being in motion parallel to the interface, is reported in another [2] of the above mentioned specialized reviews. The same particular case but for light has been treated by Saca [3]. Works dealing with light waves are taken into account in this literature survey for two reasons: (i) had the general problem been solved for relativistic waves, our case for sound would be a simple, particular case of it; (ii) when the fluids are at rest and the interface in motion (see our Sec. III B) our treatment is valid for relativistic waves, as well.

Ostashev in a rather comprehensive series of publications [4], derived the refraction law for a sound ray in a stratified moving atmosphere, where the movement, though, is of the stratified medium as a whole and not of a layer relative to an interface. At the other end of the problem, Ostashev considered sound propagation and scattering in randomly nonuniform or turbulent media [5]. Again, the case of a stratified medium whose layers move relative to one another and relative to their interface escaped his consideration and in fact this case is not examined in his most recent and wide-coverage reviews [6].

Doak [7] also considered acoustic equations in moving fluids, putting into evidence the fundamental nature of the convective, refractive, diffractive, and diffusive effects of the fluid motion and thermal inhomogeneity on the acoustic motion. It may be thought that convective motion includes the shift of one layer relative to the other as a particularly simple case, however, the equations Doak writes are very general; they include both coherent and turbulent motion, and they are not in general amenable to analytic solution.

Yeh dealt with reflection and transmission of sound waves by a moving fluid layer [8] and with reflection from a dielectric-coated moving mirror [9]. The relative movement of layer and interface was not considered in his treatment.

Lyamshev [10] took into account an interface between moving media in the particular case of media sliding parallel to the interface and only to calculate the reflection and transmission coefficients. Metz [11] observed that the deviation of waves due to the movement of the propagation media, as obtained by geometrical methods, applies to waves bound to the medium, like mechanical and ultrasound waves, and the results are not experimentally verified for light.

A theory of geometrical optics in moving dispersive media, including the laws of reflection and refraction when two media slide past one another with no intervening vacuum, was set up by Synge [12], in the frame of the special theory of relativity, reformulating the Hamilton's method in the Minkowski four-dimensional space-time instead of in three-dimensional Euclidean space. However, Synge limited his work to the particular case in which the observer is at rest relative to the interface, as in the case of Morse and Uno Ingard [2], except that Synge treats the relativistic case. Another limitation of Synge's work is that the velocities of the two media must be equal and opposite. He also determined the Lagrangian function for an isotropic medium in *general* motion, but the restriction of this general motion to the case of an isotropic medium (there are no interfaces) rules out refraction, which is the object of our interest.

Eropkin [13] treated the problem of refraction across a moving interface when the first medium is vacuum and the second medium is at rest. Actually he stated that the second medium moved rigidly with the interface, but we show in Sec. V that his starting expression is pertinent only when the second medium is at rest.

Picht [14] treated the reflection and refraction of optical waves traveling *in vacuo* at the surface of a moving medium. His procedure and results will be discussed and compared with ours in our Sec. V, where the coincidence of Picht's results with our Eq. (13) when $c_{01}=c$ (see Sec. III) will also be put in evidence.

The Arago experiment has been reexamined by Spavieri *et al.* [15] but without taking into account the motion of the lens with respect to the ether. Mamone-Capria and Pambianco [16] have recently treated in a rigorous way the nonrelativistic interpretation of the Michelson-Morley experiment, considering the reflection on a mobile mirror. However, this is the case of reflection, moreover restricted to an ether at rest with the observer.

III. REFRACTION

We obtain refraction by the Huygens construction, i.e., by the envelope of the refracted (or reflected) waves. In order to perform this construction, the equiphase surfaces (or wave fronts) in the first medium have to be perpendicular to their velocities. This occurs only in the reference system S_0 at rest with the first medium. To have neglected this fact has led Fahy [17] into error (corrected by Cavalleri *et al.* [18]).

We find $\cos\theta_2$ of the refracted angle in three steps. In the first one we pass from the laboratory system S to the system S_0 at rest with the first fluid. In the second step we perform the Huygens construction in S_0 assuming the second medium 2 at rest with medium 1. In the third step we consider the actual velocity $\mathbf{u}_2$ of medium 2 and we pass again to the laboratory system S.

A clarification is needed for the interface σ that can be a generic surface with a regular motion (i.e., without discontinuities). Locally, to find the refraction of a narrow wave beam (or ray), we can consider it as a plane (a small portion of the tangent plane). The two limitations (regular motion and narrow beam) imply that the two points A and D of Fig. 1 have infinitesimal differences of velocity. For example, if σ is rotating with the center of rotation between A and D, the velocities of both A and D are infinitesimal, i.e., σ is considered *locally* at rest. For a large beam, we divide it into narrow beams and calculate separately the refraction for each of them, with their *local* velocity $\mathbf{V}$ for σ.

A. First step

If $\mathbf{u}_1$ is the velocity of the first fluid (through which the incoming wave is propagating before refraction) with respect to the laboratory system S, we have to consider the wave in the system S_0 at rest with the fluid 1. Let $\mathbf{c}_1$ and $\mathbf{c}_{01}$ be the wave velocities in S and S_0, respectively, related to each other by

$$\mathbf{c}_1=\mathbf{c}_{01}+\mathbf{u}_1 . \tag{1}$$

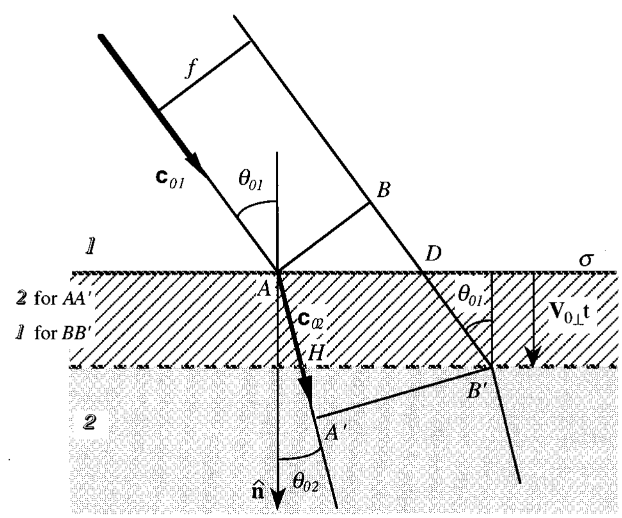

FIG. 1. A nonrelativistic wave has velocity $\mathbf{c}_{01}$ in medium 1 and equiphase surface AB perpendicular to $\mathbf{c}_{01}$ if the observer S_0 is at rest with medium 1. An interface σ having local velocity $\mathbf{V}_0$ separates medium 1 from medium 2. At time 0 medium 2 includes both the streaked region and the shaded region. At time t, when B reaches B', medium 2 is represented by the shaded area only. The streaked region is that swept by the interface σ from time 0 to time t. (Obviously, if $V_0=V_{0\perp}=0$ the streaked region is absent and $D=B'$). Consequently, during the time interval between 0 and t ray AA' travels only in medium 2 while ray BB' travels only in medium 1. $\hat{\mathbf{n}}$ is the unit vector perpendicular to the interface, chosen so that $\mathbf{c}_{01}\cdot\hat{\mathbf{n}}>0$. When a wave ray impinges on the interface in A the wave is refracted in medium 2 with velocity $\mathbf{c}_{02}$. Point B of the wave front reaches the moving interface in B', while point A reaches A' in medium 2 at rest with S_0 so that the equiphase surface $A'B'$ is still perpendicular to the refracted ray AHA'. This is the Huygens construction for media at rest but moving interface, $A'B'$ being the envelope of the spherical waves radiated by the points of the interface consecutively reached by the impinging wave front. In the general case of media moving with velocities $\mathbf{u}_1$ and $\mathbf{u}_2$, respectively, we add $\mathbf{u}_1$ to $\mathbf{c}_{01}$ and $\mathbf{u}_2$ to $\mathbf{c}_{02}$.

Take the unit vector $\hat{\mathbf{n}}$ perpendicular to the mobile interface and directed so as $\mathbf{c}_{01}\cdot\hat{\mathbf{n}}>0$. Then the incident angles θ_{01} and θ_1 in S_0 and S, respectively, are given by

$$\cos\theta_{01}=\mathbf{c}_{01}\cdot\hat{\mathbf{n}}/c_{01} \tag{2}$$

and

$$\cos\theta_1=\mathbf{c}_1\cdot\hat{\mathbf{n}}/c_1=(c_{01}\cos\theta_{01}+\mathbf{u}_1\cdot\hat{\mathbf{n}})/c_1 . \tag{3}$$

Notice that c_{01} is the speed of the wave in the fluid at rest.

B. Second step

We perform the Huygens construction in S_0 considering the fluid in the second medium at rest with the first one so that the reference system S_0 is at rest with both fluids. The situation of two fluids at relative rest in spite of the fact that their boundary plane moves is theoretical and useful as an intermediate step to find the final solution of the real problem by adding (in the third step) the relative velocity $\mathbf{u}_2-\mathbf{u}_1$ to the velocity of the refracted wave.

We plot in Fig. 1 the Huygens construction in S_0 where the local velocity of the interface σ (i.e., of a small portion of it where the wave beam impinges) is $\mathbf{V}_0=\mathbf{V}-\mathbf{u}_1$ (we denote by $\mathbf{V}$ its velocity in the laboratory system S). What is effective is the component

$$V_{0\perp}=\mathbf{V}_0\cdot\hat{\mathbf{n}} \tag{4}$$

of V_0 along the normal to the interface.

The unit vector $\hat{\mathbf{n}}$ is drawn so that $\mathbf{c}_{01}\cdot\hat{\mathbf{n}}>0$. Media 1 and 2 contain the incident and refracted wave, respectively. If σ were at rest there would be no ambiguity about which one is the incident wave. However, if $V_{0\perp}>c_{01}\cos\theta_{01}$ it is the interface σ that reaches the fleeing wave and we have to exchange medium 1 for 2 in Fig. 1. Consequently, if σ is at rest, medium 1 is always that *not* containing $\hat{\mathbf{n}}$ (drawn starting from the interface). If σ is in motion, medium 1 is that not containing $\hat{\mathbf{n}}$ only if

$$s=\mathrm{sgn}(c_{01}\cos\theta_{01}-V_{0\perp}) \tag{5}$$

is plus, medium 1 is that containing $\hat{\mathbf{n}}$ if s is minus.

We consider the first case, i.e., $s=+$, in Fig. 1, where the Huygens construction is plotted with respect to observer S_0. AB is the trace of the equiphase front in medium 1 at rest (i.e., observed in system S_0) so that it is perpendicular to the velocity $\mathbf{c}_{01}$. The wave ray which impinges in A on the boundary plane begins to travel in medium 2 with the velocity $\mathbf{c}_{02}$ along AA'. At time t when the phase front reaches A', the same phase started in B at time $t_0=0$ reaches the moving boundary plane in B'. Since the ray section BB' has always been in medium 1 and the ray section AA' in medium 2, it is

$$t=|AA'|/c_{02}=|BB'|/c_{01} . \tag{6}$$

The refracted phase front $A'B'$ is perpendicular to AA' because in this second step of our solution the medium 2 is still considered at rest with S_0. This phase front is obtained as the envelope of the spherical waves radiated by each point of the boundary plane reached by the incoming wave.

We see from Fig. 1 that $|BB'|=c_{01}t$ may also be written as

$$c_{01}t=|BD|+|DB'|=|AD|\sin\theta_{01}+V_{0\perp}t/\cos\theta_{01} . \tag{7}$$

Similarly we may write $|AA'|=c_{02}t$ as

$$c_{02}t=|A'H|+|HA|=|B'H|\sin\theta_{02}+V_{0\perp}t/\cos\theta_{02} , \tag{8}$$

where

$$|B'H|=|AD|+V_{0\perp}t(\tan\theta_{01}-\tan\theta_{02}). \tag{9}$$

Obtaining t from Eq. (7) and substituting it in Eq. (8) where Eq. (9) is used, gives, after simplifying the factor $|AD|$ that appears in both sides,

$$c_{02}\sin\theta_{01}=\sin\theta_{02}\left(c_{01}-\frac{V_{0\perp}}{\cos\theta_{01}}\right)+V_{0\perp}\sin\theta_{01}\sin\theta_{02}\left(\frac{\sin\theta_{01}}{\cos\theta_{01}}-\frac{\sin\theta_{02}}{\cos\theta_{02}}\right)+V_{0\perp}\frac{\sin\theta_{01}}{\cos\theta_{02}} . \tag{10}$$

Simplifying Eq. (10), and calling

$$m=V_{0\perp}\sin\theta_{01};\quad p=c_{01}-V_{0\perp}\cos\theta_{01};\quad q=c_{02}\sin\theta_{01}, \tag{11}$$

we obtain

$$m\ \cos\theta_{02}+p\ \sin\theta_{02}=q, \tag{12}$$

whose solution is

$$\cos\theta_{02}=\frac{mq\pm|p|(m^2+p^2-q^2)^{1/2}}{m^2+p^2} . \tag{13}$$

If $V_{0\perp}=-|V_{0\perp}|$ all the preceding expressions keep their validity.

The same Eq. (13) is obtained in the second case ($s=-$), represented by Fig. 2. In fact, Eqs. (7) and (8) become, respectively,

$$c_{01}t=(V_{0\perp}t/\cos\theta_{01})-|AD|\sin\theta_{01} , \tag{7'}$$

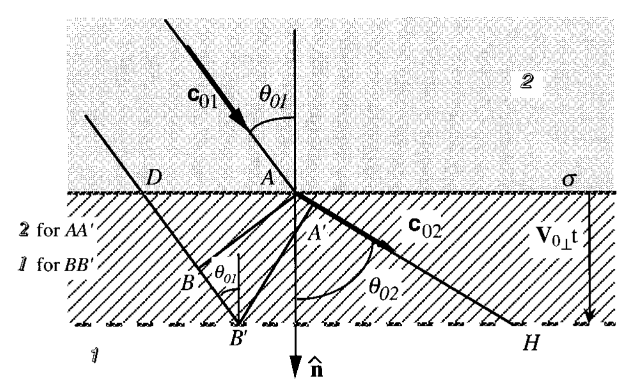

FIG. 2. Second case considered right after Eq. (13), i.e., $s=-$, where s is given by Eq. (5). Differently from Fig. 1, now medium 1 is that containing $\hat{\mathbf{n}}$ drawn from the interface σ since $V_{0\perp}>c_{01}\cos\theta_{01}$, i.e., σ reaches the fleeing wave.

and

$$c_{02}t=(V_{0\perp}t/\cos\theta_{02})-|B'H|\sin\theta_{02}, \tag{8'}$$

where

$$|B'H|=|AD|+V_{0\perp}t(\tan\theta_{02}-\tan\theta_{01}). \tag{9'}$$

Obtaining t from Eq. (7′) and substituting it in Eq. (8′) where Eq. (9′) is used, gives again Eq. (10), whence Eq. (13).

To choose the sign in Eq. (13), we observe that the solution represented by it is valid for any couple of values for c_{01} and c_{02}, therefore, when $c_{01}=c_{02}$ as well. In this case $\cos\theta_{02}=\cos\theta_{01}$ and Eq. (13) becomes

$$\cos\theta_{02}=\frac{c_{01}V_{0\perp}\sin^2\theta_{01}\pm|(c_{01}-V_{0\perp}\cos\theta_{01})(c_{01}\cos\theta_{01}-V_{0\perp})|}{c_{01}^2-2c_{01}V_{0\perp}\cos\theta_{01}+V_{0\perp}^2}. \tag{14}$$

We then must further distinguish the following cases.

(i) $s=+$, where s is given by Eq. (5), whose case coincides with the first case considered right after Eq. (5) and shown in Fig. 1. Since

$$1\geqslant\cos\theta_{01}>V_{0\perp}/c_{01}\geqslant V_{0\perp}c_{01}^{-1}\cos\theta_{01},$$

also $p=c_{01}-V_{0\perp}\cos\theta_{01}>0$. We may eliminate the absolute value sign in Eq. (14) and we see that $\cos\theta_{02}=\cos\theta_{01}$ implies that the plus has to be chosen in Eq. (14).

The second case, mentioned right after Eq. (13) and shown in Fig. 2, splits into two subcases.

(ii) $s=-$ and $c_{01}>V_{0\perp}\cos\theta_{01}$, i.e., $p>0$ with p defined in Eq. (11). In this subcase, eliminating the absolute value sign in Eq. (14) implies $|c_{01}\cos\theta_{01}-V_{0\perp}|=V_{0\perp}-c_{01}\cos\theta_{01}$ and the sign minus has to be chosen in order to get $\cos\theta_{02}=\cos\theta_{01}$.

(iii) $s=-$ and $c_{01}<V_{0\perp}\cos\theta_{01}$, i.e., $p<0$. In this subcase we may directly eliminate the absolute value sign and we must choose the sign plus in Eq. (14) to get $\cos\theta_{01}=\cos\theta_{02}$ [as in case (i)].

We may therefore synthesize the above case and subcases in the single equation

$$\cos\theta_{02}=\frac{mq+sp(m^2+p^2-q^2)^{1/2}}{m^2+p^2}, \tag{15}$$

with s given by Eq. (5).

C. Third step

We now introduce the third step of our procedure: since the medium 2 moves with velocity $\mathbf{u}_2-\mathbf{u}_1$ in S_0, the velocity $\mathbf{c}_2^*$ of the wave in medium 2 (measured in S_0) is

$$\mathbf{c}_2^*=\mathbf{c}_{02}+\mathbf{u}_2-\mathbf{u}_1. \tag{16}$$

In the laboratory system S the velocity $\mathbf{c}_2$ of the wave in medium 2 is obtained by the velocity composition from S_0 to the laboratory system S:

$$\mathbf{c}_2=\mathbf{c}_2^*+\mathbf{u}_1=\mathbf{c}_{02}+\mathbf{u}_2. \tag{17}$$

Consequently, taking the scalar product by $\hat{\mathbf{n}}$ of the first and third side of Eq. (17) gives, since $\mathbf{c}_2\cdot\hat{\mathbf{n}}=c_2\cos\theta_2$,

$$\cos\theta_2=(c_{02}\cos\theta_{02}+\mathbf{u}_2\cdot\hat{\mathbf{n}})/c_2, \tag{18}$$

where $c_2=|\mathbf{c}_{02}+\mathbf{u}_2|$.

Summarizing, if we express all the quantities in the laboratory system S it is

$$\cos\theta_2=\hat{\mathbf{c}}_2\cdot\hat{\mathbf{n}}=\frac{1}{c_2}\left\{\mathbf{u}_2\cdot\hat{\mathbf{n}}+c_{02}\frac{mq+sp(m^2+p^2-q^2)^{1/2}}{m^2+p^2}\right\}, \tag{19}$$

where c_{02} is the known speed of the wave in the second fluid when at rest. Moreover, transforming the relationships (11) to the laboratory system S and writing them in vector form, we have

$$m=(\mathbf{V}-\mathbf{u}_1)\cdot\hat{\mathbf{n}}\left\{1-\left[\frac{(\mathbf{c}_1-\mathbf{u}_1)\cdot\hat{\mathbf{n}}}{|\mathbf{c}_1-\mathbf{u}_1|}\right]^2\right\}^{1/2}, \tag{20}$$

$$p=c_{01}-(\mathbf{V}-\mathbf{u}_1)\cdot\hat{\mathbf{n}}\frac{(\mathbf{c}_1-\mathbf{u}_1)\cdot\hat{\mathbf{n}}}{|\mathbf{c}_1-\mathbf{u}_1|}, \tag{21}$$

$$q=c_{02}\left\{1-\left[\frac{(\mathbf{c}_1-\mathbf{u}_1)\cdot\hat{\mathbf{n}}}{|\mathbf{c}_1-\mathbf{u}_1|}\right]^2\right\}^{1/2}. \tag{22}$$

When $V=u_1=u_2=0$ Eq. (19) reduces to Snell's law $c_2\sin\theta_1=c_1\sin\theta_2$.

IV. REFLECTION

Reflection on a mobile surface may be treated in a similar way with the obvious simplifications $\mathbf{u}_1=\mathbf{u}_2=\mathbf{u}$, $c_1=c_2$, $c_{01}=c_{02}=c_0$, and $|AA'|=|BB'|=a=c_0t$ (see Fig. 3). We take the same sign conventions as in the case of refraction, choosing $\hat{\mathbf{n}}$ so that $\cos\theta_{01}=\mathbf{c}_{01}\cdot\hat{\mathbf{n}}/c_0>0$. Consequently, we still get Eqs. (11) and (12). The only difference with respect to refraction is that the opposite sign has to be chosen in Eq. (13). Moreover, since $c_{01}=c_{02}=c_0$, Eq. (13) reduces to Eq. (14). The final solution for the reflected angle θ_{r2} is therefore

$$\cos\theta_{r2}=\hat{\mathbf{c}}_2\cdot\hat{\mathbf{n}}=\frac{1}{c_1}\left\{\mathbf{u}\cdot\hat{\mathbf{n}}+c_0\frac{mq-p(c_{01}\cos\theta_{01}-V_{0\perp})}{p^2+m^2}\right\}. \tag{23}$$

For $\mathbf{u}=0$ Eq. (23) reduces to that of Mamone-Capria and Pambianco [16] and for $u=V=0$ to $\cos\theta_{r2}=-\cos\theta_{r1}$ (usual reflection on a fixed surface).

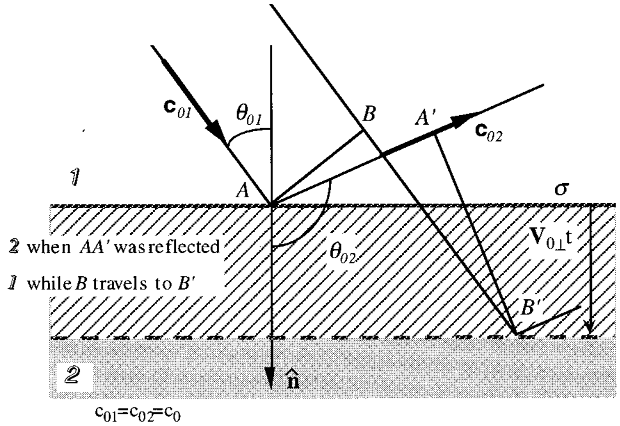

FIG. 3. Ray with nonrelativistic velocity $\mathbf{c}_{01}$ in a medium at rest with observer S_0 impinges in A on a mirror moving with velocity $\mathbf{V}_0$. The reflected wave is obtained by the Huygens construction as the envelope of the spherical waves radiated by the points of the mirror consecutively reached by the impinging wave. Since the medium is at rest with S_0 the wave front AB is perpendicular to $\mathbf{c}_{01}$ and the reflected wave front $A'B'$ is perpendicular to its velocity $\mathbf{c}_{02}$. The streaked region is that swept by the mirror σ during the time interval between 0 and t. The shaded region is that beyond the mirror at time t.

V. DISCUSSION AND COMPARISONS

It might be argued that nonrelativistic waves are only sound waves, and therefore, our theory cannot be applied to the interpretation of light or e.m. wave experiments. This possible objection could arise from the fact that we are used to consider light transmission in media with refraction index $n \leqslant 2$. However, for the sake of generality our results could well apply to e.m. transmission near absorption frequencies, where $n \geqslant 1$. Moreover, our second step (Sec. III B) is valid even for a relativistic value of V.

We now compare our results with those of the works that approached the problem, giving relevant contributions. When $c_{01}=c_1$ our Eq. (13) reduces to that found by Picht [14] who, however, treated only the case of light. Indeed, for light, the case $c_{01}=c$ means that the first medium is vacuum. This is why Picht needs not pass from observer S to S_0 at rest with the first fluid. Moreover, Picht disregards the sign problem, i.e., he does not introduce the s given by our Eq. (5), and this procedure leads to a correct result only in case (i) of our Sec. III B but not in subcases (ii) and (iii).

Notice that Eropkin's Eq. (15) is valid when both fluids are at rest, and only their interface is in motion (normally to itself) as can clearly be seen by Eq. (9) of Eropkin. Looking at Eropkin's Fig. 2, denoting by t the time for the light ray to go from the starting point A to the interface and by t_2 the time for the light ray to travel from the interface to the arrival point B in the second medium, we have $s_1=ct_1$ (the first medium is vacuum), $s_2=c_{02}t_2=ct_2/n$ (if the second medium is at rest), i.e., $ns_2=ct_2$. Adding the two equations we get $s_1+ns_2=c(t_1+t_2)=ct$, which is Eq. (9) of Eropkin. We clearly see that his equation is valid when both media are at rest and only the interface is in motion, contrary to the statement at the beginning of his Sec. 3. Consequently, he has treated the problem only under the condition of our second step (our Sec. III B).

VI. CONCLUSIONS

We have thus solved the general problem of refraction in the nonrelativistic approximation when the two media and the interface are moving. The solution for the cosine of the refracted wave is given by Eq. (19), valid when $|\mathbf{u}_1|$ and $|\mathbf{u}_2|$ are small compared to the speed of light c. The velocity $V=\beta c$ of the interface can, on the contrary, be comparable with c. In such a case our treatment implies that one knows the correct direction of $\hat{\mathbf{n}}$ in the system S_0 at rest with fluid 1. The same considerations hold for reflection whose solution is given by Eq. (23). Previous works [13,14] have only obtained our second step (Sec. III B). Moreover, one of them [13] is only in implicit form, and the other [14] is in explicit form but disregards the sign s given by Eq. (5).

Nevertheless it is interesting that Eropkin [13] had obtained our implicit Eq. (12) starting from the Fermat principle, while we started from the Huygens construction. Notice that so far Fermat's principle had been theoretically proved only for fluids and interface at rest. The coincidence of the two results now proves that Fermat's principle is valid even when the interface is in motion, if the fluids are at rest.

[1] See for instance, A. Sommerfeld, *Mechanics of Deformable Bodies* (Academic, New York, 1950), Chap. V; L. Landau and E. Lifchitz, *Mecanique des Fluides* (Mir, Moscow, 1971), Sec. 67.

[2] P. M. Morse and K. Uno Ingard, in *Encyclopaedia of Physics*, edited by S. Flügge, Acoustics I (Springer-Verlag, Berlin 1961), Vol. XI/1, Sec. 5, p. 11.

[3] J. M. Saca, Proc. IEEE (USA) **68**, 409 (1980).

[4] V. E. Ostashev, Izv. Atmos. Oceanic Phys. **21**, 274 (1985); **22**, 936 (1986); **23**, 370 (1987); **27**, 956 (1991); Sov. Phys. Acoust. **31**, 130 (1985); **32**, 303 (1986); **34**, 402 (1988).

[5] V. E. Ostashev, Atmos. Opt. **4** (9), 653 (1991); *V International Symposium on Long Range Sound Propagation, Milton Keynes, England*, edited by K. Attenborough (The Open University, Milton Keynes, 1992), p. 203.

[6] V. E. Ostashev, Izv. Atmos. Oceanic Phys. **25**, 661 (1989), and (unpublished).

[7] P. E. Doak (unpublished).

[8] C. Yeh, J. Acoust. Soc. Am. **4**, 217 (1966).

[9] C. Yeh, J. Opt. Soc. Am. **57**, 657 (1967).

[10] L. M. Lyamshev, Sov. Phys. Acoust. **10**, 205 (1964).

[11] A. Metz, C. R. Acad. Sci. II **248**, 1615 (1959).

[12] J. L. Synge, Communications of the Dublin Institute for Advanced Studies, Series A, N.12, 1956.

[13] D. J. Eropkin, Z. Phys. **58**, 268 (1930).

[14] J. Picht, Z. Phys. **58**, 667 (1930).

[15] G. Spavieri and G. Contreras, Nuovo Cimento B **91**, 143 (1986); G. Spavieri, Phys. Lett. **1**, 373 (1988); G. Spavieri and G. Bergamaschi, Nuovo Cimento B **104**, 497 (1989).
[16] M. Mamone-Capria and F. Pambianco, Phys. Found. **24**, 885 (1994).
[17] E. F. Fahy, Nature **188**, 396 (1960).
[18] G. Cavalleri, L. Galgani, G. Spavieri, and G. Spinelli, Lett. Nuovo Cimento **17**, 25 (1976).